

PreCalculus

Faige Kerner

Topic: Ellipses

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad C(h, k)$$

a = distance from center to vertex on the ~~major axis~~
 b = " " " " " " minor axis

a is always under the major axis

Major axis

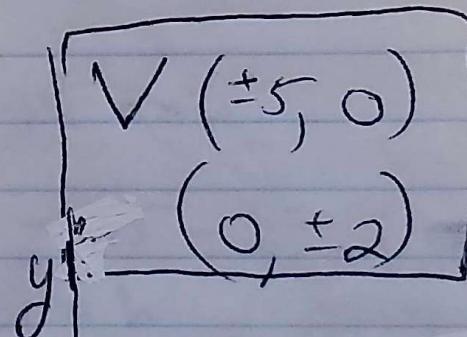
Minor axis

Focus (pl. foci) always lie on the major axis

c = distance from the center to the focus

$$c^2 = a^2 - b^2 \quad \leftarrow \text{to find } c,$$

$$\textcircled{1} \quad \frac{x^2}{25} + \frac{y^2}{4} = 1$$

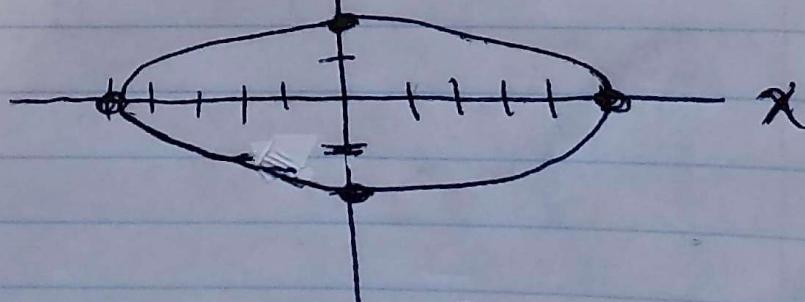


$$a = 5$$

$$b = 2$$

$$c^2 = 25 - 4$$

$$c = \sqrt{21}$$



Focus - That point that the distance from focus to any pt on the ellipse, has the same sum

PreCalculus

Fangie Krueger

$$(2) \frac{x^2}{64} + \frac{y^2}{100} = 1$$

C (0, 0)

V ($\pm 8, 0$)

(0, ± 10)

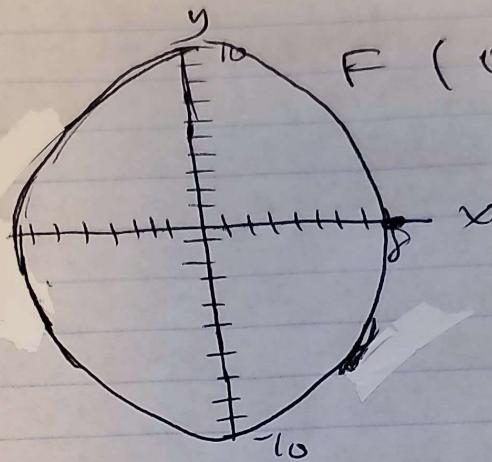
F (0, ± 6)

$$a = 10$$

$$b = 8$$

$$c^2 = 100 - 64$$

$$c = 6$$



- (3) Find an equation of an ellipse whose center is at origin, one vertex at (0, 5) one focus at (0, 2). Find all vertices + sketch.

(0, 2) means y is the major axis

$$(0, 5) \Rightarrow a = 5$$

$$4 = 25 - b^2$$

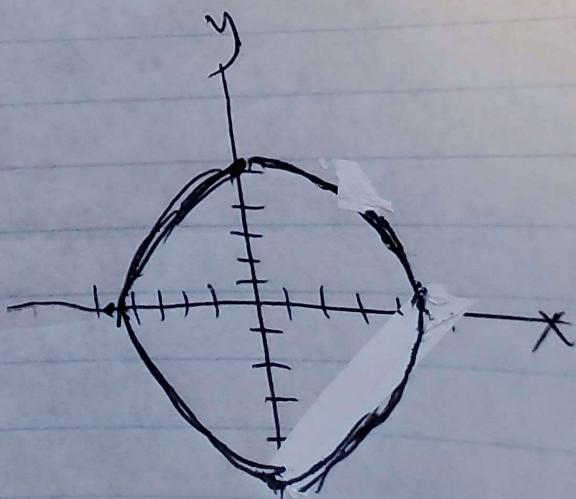
$$c = 2$$

$$\sqrt{21} = b$$

$$\boxed{\frac{x^2}{21} + \frac{y^2}{25} = 1}$$

V (0, ± 5) C(0,0)

($\pm \sqrt{21}, 0$) F (0, ± 2)



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PreCalculus

Faige Kramer

Class 228-29 / 2, 12, 14, 18, 20, 24, 27, 30, 33

Homework 228-9 / 3, 5, 13, 15, 19, 25, 26, 32

- (Hint: Suppose the ellipse has equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$. Then the vertices are at $(\pm a, 0)$ and the foci are at $(\pm c, 0)$, where $c = \sqrt{a^2 - b^2}$.)*
- d. an equation of the ellipse. $\frac{x^2}{25} + \frac{y^2}{169} = 1$.
 2. An ellipse has equation $\frac{x^2}{25} + \frac{y^2}{169} = 1$.
 - a. Is its major axis horizontal or vertical? Vert.
 - b. Find the coordinates of its vertices and foci. $V(0, \pm 13)$, $F(0, \pm 12)$
 3. Describe the graphs of $\frac{x^2}{9} + y^2 < 1$ and $\frac{x^2}{9} + y^2 > 1$.
 4. **Discussion** If F_1 and F_2 are fixed points in space, describe the set of points P such that (a) $PF_1 + PF_2 = 8$ and (b) $PF_1 + PF_2 < 8$.

WRITTEN EXERCISES

Sketch each ellipse. Find the coordinates of its vertices and foci.

A 1. $\frac{x^2}{36} + \frac{y^2}{16} = 1$

2. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

3. $\frac{x^2}{16} + \frac{y^2}{25} = 1$

4. $4x^2 + 25y^2 = 100$

5. $9x^2 + 25y^2 = 225$

6. $6.25x^2 + 4y^2 = 25$

7. Sketch the graph of each inequality: (a) $\frac{x^2}{25} + \frac{y^2}{9} \leq 1$ (b) $\frac{x^2}{4} + \frac{y^2}{16} \geq 1$

 Find the domain and range of each function. Then graph the function. You may find a graphing calculator helpful.

8. $y = 3\sqrt{1 - x^2}$

9. $y = -\frac{1}{3}\sqrt{36 - x^2}$

10. a. On a single set of axes, graph $x^2 + y^2 = 1$ and $\left(\frac{x}{3}\right)^2 + y^2 = 1$.

b. Give the area of the circle and guess the area of the ellipse.

11. a. On a single set of axes, graph $x^2 + y^2 = 1$ and $x^2 + \left(\frac{y}{2}\right)^2 = 1$.

b. Give the area of the circle and guess the area of the ellipse.

Each ellipse has its center at the origin. Find an equation of the ellipse.

12. Vertex $(7, 0)$; minor axis 2 units long

13. Vertex $(0, -9)$; minor axis 6 units long

14. Vertex $(0, -13)$; focus $(0, -5)$

15. Vertex $(17, 0)$; focus $(8, 0)$

12. $\frac{x^2}{49} + y^2 = 1$ 13. $\frac{x^2}{9} + \frac{y^2}{81} = 1$ 14. $\frac{x^2}{144} + \frac{y^2}{169} = 1$ 15. $\frac{x^2}{289} + \frac{y^2}{225} = 1$

tractor to show that the angle between $\overline{F_1P}$ and the tangent is congruent to the angle between $\overline{F_2P}$ and the tangent. Repeat the experiment for another point P .

b. What property of the ellipse does part (a) illustrate?

17. **Investigation** Refer to the Activity on page 226. What happens to an ellipse as its foci F_1 and F_2 move toward each other? If F_1 and F_2 coincide? The ellipse becomes more circular. When $F_1 = F_2$, the figure is a circle.

Sketch the graphs of the given equations on a single set of axes. Then determine algebraically where the graphs intersect.

18. $9x^2 + 2y^2 = 18$ $(0, -3)$, $\left(-\frac{4}{3}, 1\right)$

19. $x^2 + 4y^2 = 400$ $(12, -8)$, $(16, -6)$
 $x - 2y = 28$

20. $2x^2 + y^2 = 9$ $(-2, 1)$

21. $x^2 + 4y^2 = 16$ $(2, \sqrt{3})$, $(-2, \sqrt{3})$
 $|x| = 2$ $(-2, -\sqrt{3})$, $(2, -\sqrt{3})$

22. a. What happens to the graph of an equation of an ellipse when x is replaced by $x - 5$? The graph is shifted 5 units to the right.

b. Graph $\frac{x^2}{4} + y^2 = 1$ and $\frac{(x - 5)^2}{4} + y^2 = 1$ on a single set of axes.

23. a. What happens to the graph of an equation of an ellipse when y is replaced by $y + 6$? The graph is shifted 6 units down.

b. Graph $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and $\frac{x^2}{4} + \frac{(y + 6)^2}{9} = 1$ on a single set of axes.

Sketch each ellipse. Label the center and vertices.

24. $\frac{(x - 3)^2}{4} + \frac{(y - 6)^2}{25} = 1$

25. $\frac{(x + 5)^2}{25} + \frac{(y - 4)^2}{16} = 1$

26. $(x + 7)^2 + \frac{(y - 5)^2}{9} = 1$

27. $4(x + 2)^2 + (y - 5)^2 = 4$

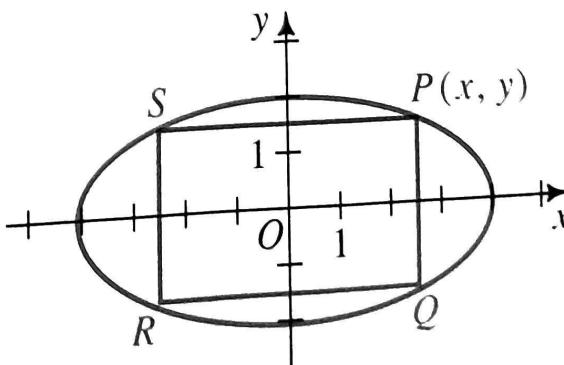
 You may find it helpful to have a graphing calculator to complete Exercise 28(b).

28. In the figure at the right, rectangle $PQRS$ is inscribed in the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

a. Show that the area of the rectangle is

$$A(x) = 4x \sqrt{4 - \frac{x^2}{4}}$$

b. Approximate



29. When the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is rotated about

either of its axes, an *ellipsoid* is formed.

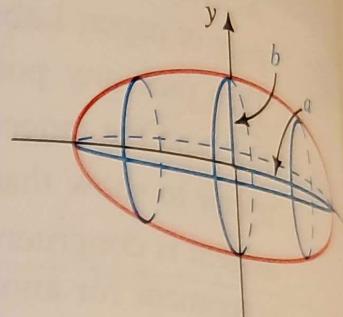
- a. The volume of the ellipsoid shown is

$$V = \frac{4}{3}\pi b^2 a. \text{ Interpret this volume if the}$$

original ellipse is a circle.

- b. Sketch the ellipsoid formed by rotating the given ellipse about its minor axis and guess

$$\text{its volume. } \frac{4}{3}\pi a^2 b$$



Sketch each ellipse. Find the coordinates of its vertices and foci.

30. $\frac{(x - 5)^2}{25} + \frac{(y + 3)^2}{9} = 1 \quad V: (0, -3), (10, -3); F: (1, -3), (9, -3)$

31. $\frac{(x + 6)^2}{12} + \frac{(y - 4)^2}{16} = 1 \quad V: (-6, 0), (-6, 8); F: (-6, 2), (-6, 6)$

32. $9(x - 3)^2 + 4(y + 5)^2 = 36 \quad V: (3, -2), (3, -8); F: (3, -5 + \sqrt{5}), (3, -5 - \sqrt{5})$

33. $(x + 1)^2 + 4(y + 3)^2 = 9 \quad V: (-4, -3), (2, -3); F: \left(-1 + \frac{3\sqrt{3}}{2}, -3\right), \left(-1 - \frac{3\sqrt{3}}{2}, -3\right)$

34. $x^2 + 25y^2 - 6x - 100y + 84 = 0$

(Hint: Complete the squares in x and in y . Begin by rewriting the equation in this form: $(x^2 - 6x + \quad) + 25(y^2 - 4y + \quad) = -84$.) See below.

35. $9x^2 + y^2 + 18x - 6y + 9 = 0 \quad V: (-1, 6), (-1, 0); F: (-1, 3 + 2\sqrt{2}), (-1, 3 - 2\sqrt{2})$

36. $9x^2 + 16y^2 - 18x - 64y - 71 = 0 \quad V: (-3, 2), (5, 2); F: (1 + \sqrt{7}, 2), (1 - \sqrt{7}, 2)$

34. $V: (-2, 2), (8, 2); F: (3 + 2\sqrt{6}, 2), (3 - 2\sqrt{6}, 2)$

 You may find a graphing calculator or graphing software helpful to complete Exercise 37.

37. A graphing calculator or computer software can be used to sketch the graph of an equation that represents a function.

a. Solve $\frac{x^2}{36} + \frac{y^2}{16} = 1$ for y . The result involves two equations. $y = \pm \frac{2}{3}\sqrt{36 - x^2}$

- b. Graph the ellipse in part (a). Does the result agree with the graph in Exercise 1? Yes

In Exercises 38–41, find an equation of the ellipse described.